Breit-Wigner Enhancement Considering the Dark Matter Kinetic Decoupling

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In the paper we study the Breit-Wigner enhancement of dark matter (DM) annihilation considering the kinetic decoupling in the evolution of DM freeze-out at the early universe. Since the DM temperature decreases much faster (as $1/R^2$) after kinetic decoupling than that in kinetic equilibrium (as $1/R$) we find the Breit-Wigner enhancement of DM annihilation rate after the kinetic decoupling will affect the DM relic density significantly. Focusing on the model parameters that trying to explain the anomalous cosmic positron/electron excesses observed by PAMELA/Fermi/ATIC we find the elastic scattering $Xf \to Xf$ is not efficient to keep dark matter in kinetic equilibrium, and the kinetic decoupling temperature T_{kd} is comparable to the chemical decoupling temperature $T_f \sim O(10)GeV$. The reduction of the relic density after T_{kd} is significant and leads to a limited enhancement factor $\sim O(10^2)$. Therefore it is difficult to explain the anomalous positron/electron excesses in cosmic rays by DM annihilation and give the correct DM relic density simultaneously in the minimal Breit-Wigner enhancement model.

I. INTRODUCTION

The recent cosmic ray observations by PAMELA[1], ATIC[2] and Fermi[3] have all reported an excess of positrons and electrons from ~ 10 GeV up to ~ 1 TeV. These anomalies have stimulated a lot of interests, especially these excesses may be attributed to the signals of dark matter annihilation in the Galaxy. If these extra positrons/electrons are indeed from DM annihilation, it requires definite properties of DM. For example, DM should annihilate into lepton final states dominantly and should have a much larger annihilation cross section ($\langle\sigma v\rangle \sim 10^{23}\mathrm{cm}^3\mathrm{s}^{-1}$) than the natural value ($\langle\sigma v\rangle \sim 10^{-26}\mathrm{cm}^3\mathrm{s}^{-1}$) at freeze out [4–6]. The annihilation cross section at freeze out determines the DM relic density if DM is generated thermally at the early universe.

In general, the DM annihilation cross section $\langle\sigma v\rangle$ depends on the averaged velocity of DM. For example, in the usual weakly interacting massive particle (WIMP) scenario, $\langle\sigma v\rangle$ can be expanded to a form of $a+b\langle v^2\rangle+O(v^4)$ at the non-relativistic limit [7]. If the annihilation process is s-wave dominant, $\langle\sigma v\rangle$ is a constant. For the p-wave annihilation, $\langle\sigma v\rangle$ is proportional to $\langle v^2\rangle$. Therefore the DM annihilation by p-wave is suppressed today than the decoupling time since the WIMP usually has a velocity of $v \sim 10^{-1}$ at the freeze-out epoch and cools when universe expands. The DM velocity near the solar system is $v \sim 10^{-3}$, much smaller than that at the decoupling epoch.

However, as indicated by the PAMELA, ATIC and Fermi data, we actually ask for a much larger annihilation cross section today to account for the excesses than that at the early universe. Contrary to the analysis before for the p-wave annihilation we require an annihilation form $\langle\sigma v\rangle$ depends on $\sim 1/v^n$. This form leads to a large annihilation cross section today with low DM velocity and explains the cosmic positron anomaly and relic density simultaneously. Some mechanisms are soon proposed to achieve this aim after these results published, such as the Sommerfeld enhancement [8, 9] and the Breit-Wigner enhancement [10–14].

For the Sommerfeld enhancement, a new light mediator with mass of O(GeV) is introduced, and provide an enhancement factor of $S \sim \pi\alpha_X/v$ (α_X is coupling constant between DM and mediator). For the Breit-Wigner enhancement, the DM annihilates via a narrow resonance, and an enhance factor of $S \sim max[\delta,\gamma]^{-1}/O(10)$ can be obtained [12] (δ, γ are defined as $\delta = (4m^2 - M^2)/4m^2$ and $\gamma = \Gamma/M$ respectively, where m is the mass of DM, M and Γ are the mass and decay width of the resonance respectively). One can achieve correct enhancement factor $S = \langle\sigma v\rangle_{T=0}/\langle\sigma v\rangle_{T\sim T_f}$ (T_f is the temperature of chemical decoupling) by adjusting the parameters appropriately.

It seems that the enhancement should not be important in the early universe when the velocity of DM is $\sim O(10^{-1})$, and the enhancement factor is only $S \sim O(1)$. However, some recent studies showed that such effects are not negligible even at the freeze-out epoch [15–18], especially for the Sommerfeld enhancement. The Ref. [17, 18] pointed out that it may be difficult to achieve the required enhancement factor in the minimal Sommerfeld models considering the effect at the early universe.

In this work, we will give a careful inspection on the Breit-Wigner mechanism at the DM freeze-

out process. For the Breit-Wigner mechanism, the DM annihilation continues after the chemical decoupling until the DM velocity drops below the cut-off scale. Therefore the relic density is determined by the cut-off scale related to δ and γ [14]. In the work we will show another important factor in determining the relic density, *i.e.* the kinetic decoupling process [21–23].

After the chemical decoupling at $x \sim 20$ (x represents the temperature of the universe which is defined as $x = m/T$), the DM particle is still kept in kinetic equilibrium via the scattering with the hot bath. When such scattering is not efficient to keep DM in kinetic equilibrium, the DM momentum is red-shifted with the scale factor R, which leads to a rapid decrease of DM temperature as $T_X \sim R^{-2}$ rather than $T_X \sim R^{-1}$ at the kinetic equilibrium epoch [21–23]. Therefore, after the kinetic decoupling $\langle \sigma v \rangle$ increases quickly and then reduces the abundances of DM more efficiently. Taking this effect into account we find the Breit-Wigner mechanism is hard to provide a self-consistent explanation for both the DM relic density and the positron anomaly today.

This paper is organized as following. In Section II, we briefly describe the Breit-Wigner enhancement mechanism at the DM freeze-out epoch. In Section III, we discuss the kinetic decoupling process. We will calculate the kinetic decoupling temperature and the DM relic density including such effect. In Section IV, we investigate the enhancement factor required by the cosmic positron measurements. We will study the parameter space and discuss whether there exists such parameters to explain all the observations. Finally we give our conclusions and discussions in Section V.

II. THE BREIT-WIGNER ENHANCEMENT

In Ref. [12], the DM annihilation process is assumed through $X\bar{X} \to R \to f\bar{f}$, where R is a narrow resonance with mass $M = \sqrt{4m^2(1-\delta)}$ and decay width $\Gamma = M\gamma$ with $|\delta|, \gamma \ll 1$. For a scalar resonance, the annihilation cross section is given as,

$$\sigma = \frac{16\pi}{M^2 \bar{\beta}_i \beta_i} \frac{\gamma^2}{(\delta + v^2/4)^2 + \gamma^2} B_i B_f, \qquad (1)$$

where $\bar{\beta}_i$ and β_i are defined as $\sqrt{1-4m^2/M^2}$ and $1-4m^2/s$ respectively, s is given by $s = (p_1+p_2)^2$, B_i and B_f denote the branching fractions of the resonance into initial and final states respectively, v is the relative velocity of two initial particles. For $\delta > 0$, there exists an un-physical pole, but $B_i/\bar{\beta}_i$ is well defined. For simplicity, we parameterize the cross section as [12]

$$\sigma v = \sigma_0 \frac{\delta^2 + \gamma^2}{(\delta + z)^2 + \gamma^2}. \qquad (2)$$

Here $\sigma_0 = \sigma v|_{T=0}$ means the cross section at zero temperature limit which is velocity independent, and is set as a free parameter in our work [1]. z is defined in the form of $s \equiv 4m^2(1+z)$ which equals $v^2/4$ in the non-relativistic limit.

In order to calculate the DM relic density, it is necessary to solve the Boltzmann equation [7]

$$\frac{dY}{dx} = -\lambda' x^{-2} \langle \sigma v \rangle (Y^2 - Y_{eq}^2) \qquad (3)$$

where $Y = n_{\rm DM}/s$ is the DM number density normalized by the entropy density s, λ' is defined as $\lambda' = \frac{s}{H}|_{x=1}$. The entropy density $s(x)$ and the Universe expansion rate $H(x)$ of the universe are given by

$$s(x) = \frac{2\pi^2 g_{*S}}{45} \frac{m^3}{x^3}\ , \ H(x) = \sqrt{\frac{4\pi^3 g_*}{45 m_{pl}^2}} \frac{m^2}{x^2}\ , \qquad (4)$$

where $g_*(g_i)$ is the effective number of degrees of freedom for radiations (DM), and g_{*S} is the effective number of degrees of freedom defined by the entropy density. The $\langle \sigma v \rangle$ can be parameterized as $\langle \sigma v \rangle = \sigma_0 x^{-n}$ and the chemical decoupling temperature is obtained as [7]

$$x_f \simeq ln\varepsilon - (n + 1/2) ln(ln\varepsilon), \qquad (5)$$

where $\varepsilon \equiv c(c+2)a\lambda$ ($c \sim 1$ is a constant), $\lambda \equiv \lambda' \sigma_0$, and $a = 0.145(g_i/g_*)$ is defined in the form of $Y_{eq} = a x^{3/2} e^{-x}$ at low temperature. The final Y as x tends to ∞ could be obtained approximately as $Y_\infty \simeq (n+1) x_f^{n+1}/\lambda$, and then the relic density $\Omega_X h^2 = 2.74 \times 10^8 \frac{m}{GeV} Y_\infty$.

In Ref. [12], after parameterizing $\langle \sigma v \rangle$ for $\delta > 0$, the Boltzmann equation could be rewritten as,

$$\frac{dY}{dx} = -\frac{\lambda}{x^2} \frac{\delta^2 + \gamma^2}{(\delta + \xi x^{-1})^2 + \gamma^2} (Y^2 - Y_{eq}^2), \qquad (6)$$

[1] For the scalar resonance discussed above, σ_0 is $\frac{32\pi B_i B_f}{M^2 \bar{\beta}_i} \frac{\gamma^2}{\delta^2+\gamma^2}$. For the Z' model in Ref. [14] , σ_0 denotes $\frac{a^2 g'^4}{16\pi m^2} \frac{1}{\delta^2+\gamma^2}$. σ_0 is a combination of δ, γ and other parameters determined by the detailed model. It is indeed a free parameter here. For more general discussions about the cross section formula of DM annihilation via s-channel resonance, see Ref. [19]

where $\xi \approx 1/\sqrt{2}$ is a constant (in fact, there is an assumption here that $x \sim v^{-2}$ or DM stays in kinetic equilibrium until very low temperature in Eq. (6)). For the Breit-Wigner enhancement, the freeze-out process begins at $\tilde{x}_f \sim O(10)$ [2], and continues until the temperature of $x_b \simeq max[\delta,\gamma]^{-1}$ when the DM annihilation cross section does not increase with the universe cooling. The final value of Y is $Y_\infty \simeq x_b/\lambda$. In the ordinary S-wave non-resonant annihilation scenario with $\langle\sigma v\rangle = constant$, one could obtain $Y_\infty \simeq x_{f0}/\lambda_0$, where $x_{f0} \sim 20$, $\lambda_0 \simeq \lambda' \times 10^{-9} GeV^{-2}$. Then the enhancement factor is achieved as $S \simeq x_b/x_{f0} \simeq max[\delta,\gamma]^{-1}/O(10)$ [12]. The Breit-Wigner enhancement has been used to explain the anomalous positron excesses which require an enhancement factor of $\sim O(10^3)$.

III. KINETIC DECOUPLING OF DM PARTICLES

In the early universe, the DM production and annihilation processes $X\bar{X} \rightleftharpoons f\bar{f}$ are efficient to keep DM particles in chemical and kinetic equilibrium. After chemical decoupling at T_f DM may keep in kinetic equilibrium by momentum exchange with the hot bath of the standard model particles via the t-channel scattering $Xf \to Xf$, until the temperature decreases to the kinetic decoupling temperature T_{kd}.

Before kinetic decoupling, the DM has the same temperature as the thermal bath. After kinetic decoupling, the temperature of DM T_X decreases as $1/R^2$, while the the temperature of thermal radiation still decrease as $1/R$. So the T_X could be determined as [21–23]

$$\left\{ \begin{array}{c} T_X = T, \quad T_X > T_{kd} \\ T_X = T^2/T_{kd}, \quad T_X \leq T_{kd} \end{array} \right. . \tag{7}$$

Since T_X is different from T one can define a parameter $x_{_X}$ related to DM temperature T_X as

$$x_{_X} = \frac{m}{T_X} = \frac{2}{v_0^2}, \tag{8}$$

where $v_{_0}$ is the most probable velocity of DM. The $\langle\sigma v\rangle$ is a function of $x_{_X}$ which is given by [11]

$$\langle\sigma v\rangle = \frac{1}{n_{EQ}^2}\frac{m}{64\pi^4 x_{_X}}\int_{4m^2}^{\infty}\hat{\sigma}(s)K_1(\frac{x_{_X}\sqrt{s}}{m})ds \ , \tag{9}$$

with

$$n_{EQ} \ = \ \frac{g_i}{2\pi^2}\frac{m^3}{x_{_X}}K_2(x_{_X}) \ , \tag{10}$$

$$\hat{\sigma}(s) \ = \ 4E_1E_2\sigma v g_i^2\sqrt{1-\frac{4m^2}{s}} \ , \tag{11}$$

where $K_1(x)$ and $K_2(x)$ are the modified Bessel functions of the first and second type respectively.

After kinetic decoupling, the temperature of DM decreases rapidly, and the Breit-Wigner enhancement increases significantly. In the Fig. 1, we show the enhancement factor of $\langle\sigma v\rangle/\langle\sigma v\rangle|_{x=20}$ for $x_{_{kd}} = 50, 10^3, 10^4, 10^5$ respectively. We also give the results in the limit of $x_{_{kd}} = \infty$ which denotes no kinetic decoupling. From Fig. 1, we can see the $\langle\sigma v\rangle$ for $x_{_{kd}} = 50$ increases more quickly reaching the maximal value than the cases without kinetic decoupling. On the other hand, for a large value of $x_{_{kd}} = 10^4, 10^5$, such effects are not very obvious. These results could be understood easily from Eq. (2) by assuming $\langle\sigma v\rangle \sim \sigma v|_{z\to v_0^2}$ roughly. When $x > x_{_{kd}}$ and $v_0^2 \gg \delta,\gamma$, $\langle\sigma v\rangle$ increase as $x^2/x_{_{kd}}$ rather than x, and reaches σ_0 more quickly for small $x_{_{kd}}$.

The Fig. 2 shows the effects of kinetic decoupling in the calculation of the relic density. After kinetic decoupling, the annihilation of DM becomes more significantly, and reduce the relic density more efficiently. If kinetic decoupling is very late $x_{_{kd}} \gg \tilde{x}_f$, for example $x_{_{kd}} = 10^4$, such effect is not very important compared with the case without kinetic decoupling. However, if the kinetic decoupling occurs at nearly the same epoch as the chemical decoupling, the efficient annihilation would reduce DM relic density by about one order of magnitude. Therefore the kinetic decoupling temperature T_{kd} is a very important parameter in the calculation of the DM relic density.

The kinetic decoupling temperature T_{kd} can be determined using the method in Refs. [18, 22]. If the momentum transfer rate drops below the expansion rate, the DM decouples from the kinetic equilibrium with the radiation background. Therefore, the T_{kd} can be determined approximately by the relation of $\Gamma_k(T_{kd}) = H(T_{kd})$. The momentum transfer rate is defined as Ref. [18, 22]

$$\Gamma_k \sim n_r\langle\sigma v\rangle_s\frac{T}{m} \tag{12}$$

where n_r is the number density of massless fermions with $n_r = \frac{3}{4}\cdot\frac{1.202}{\pi^2}g_fT^3$, $\langle\sigma v\rangle_k$ is the thermally averaged cross section for the scattering process $Xf \to Xf$. Note that there exists a factor of T/m in the above formula, which reflects the approximate momentum transfer at each collision.

[2] The $\tilde{x}_f$ could be achieved approximately by setting $\lambda \to \lambda(\delta^2+\gamma^2)/\xi$ and $n \to -2$ in the Eq. (5).

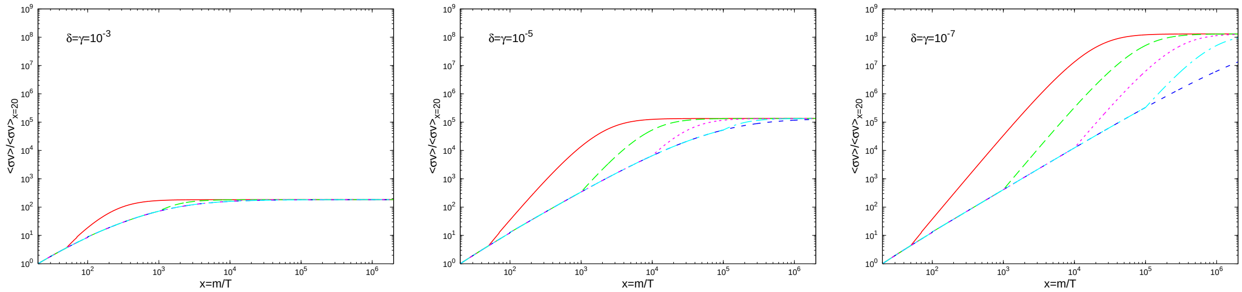

FIG. 1: The Breit-Wigner enhanced relative cross section $\langle\sigma v\rangle/\langle\sigma v\rangle|_{x=20}$ as a function of x. The curves in the figures from left to right denote $x_{kd} = 50, 10^3, 10^4, 10^5, \infty$ respectively. The model parameters in the figures from left to right are set as $\gamma = \delta = 10^{-2}, 10^{-5}, 10^{-7}$ respectively.

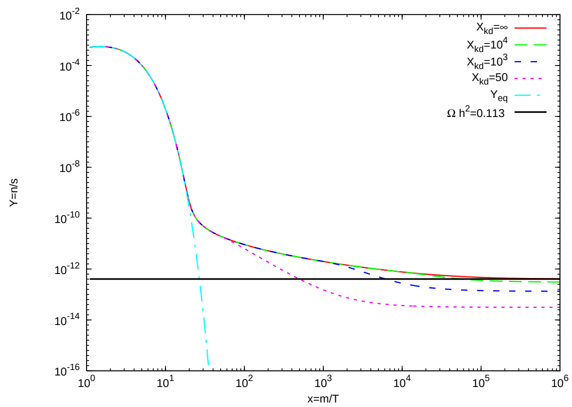

FIG. 2: The evolution of DM abundance Y as a function of x. The x_{kd} are taken as $50, 10^3, 10^4, \infty$ respectively.

Since the elastic scattering $Xf \to Xf$ via t-channel is suppressed by the propagator of R with $1/(t-M^2)^2 \sim 1/M^4$, the cross section of this process is much smaller than the annihilation cross section. The explicit formula of the cross section for $Xf \to Xf$ depends on the model details. An approximate cross section of $Xf \to Xf$ is related to $X\bar{X} \to f\bar{f}$ as

$$\sigma v_s \sim a_s \sigma_0 (\delta^2 + \gamma^2) \frac{T^2}{m^2} \ , \qquad (13)$$

where $a_s \le O(1)$ is a constant determined by the form of the interaction (for more details, see the appendix). Then we can estimate T_{kd} by setting $\Gamma_k(T_{kd}) = H(T_{kd})$. Then we get

$$\begin{aligned} T_{kd} &\sim 2.0 \left[\frac{\sqrt{g_*} m^3}{g_f a_s \sigma_0 (\delta^2+\gamma^2) m_{pl}} \right]^{\frac{1}{4}} \\ &\sim 30.GeV \left[\frac{1}{a_s}\right]^{\frac{1}{4}} \left[\frac{10^{-6} GeV^{-2}}{\sigma_0}\right]^{\frac{1}{4}} \left[\frac{10^{-9}}{\delta^2+\gamma^2}\right]^{\frac{1}{4}} \\ &\times \left[\frac{4}{g_f}\right]^{-\frac{1}{4}} \left[\frac{g_*}{100}\right]^{\frac{1}{8}} \left[\frac{m}{1TeV}\right]^{\frac{3}{4}} \ . \end{aligned} \qquad (14)$$

From above estimation, we can see the typical T_{kd} in the Breit-Weigner enhancement model is O(10)GeV, which is much larger than that in the ordinary WIMP model. For example, the T_{kd} for neutralino in the SUSY model is only O(10)MeV [21, 22].

If T_{kd} in Eq. (14) is larger than the DM freeze-out temprature[3] $T_f \sim m/\tilde{x}_f \sim m/20$, it means the elastic scattering becomes unimportant before the chemical decoupling. However, the DM particles are still kept in thermal equilibrium by the annihilation process $f\bar{f} \to XX$. Therefore T_{kd} should be defined as $min(T_f, T'_{kd})$, where T'_{kd} is determined by the elastic scattering as given in Eq. (14).

For a more precise calculation, one need to derive the DM temperature $T_X(T)$ from the Boltzmann equation

$$L[f] = C[f] \ , \qquad (15)$$

where the L and C are Liouville operator and collision operator for the scattering process respectively. A general relation between the T_X and T has been provided by Ref. [23]. In our work, we still adopt the simple relation between the T_X

[3] Here we define $\tilde{x}_f$ as the time when $n_{DM}(\tilde{x}_f) = 10 n^{eq}_{DM}(\tilde{x}_f)$.

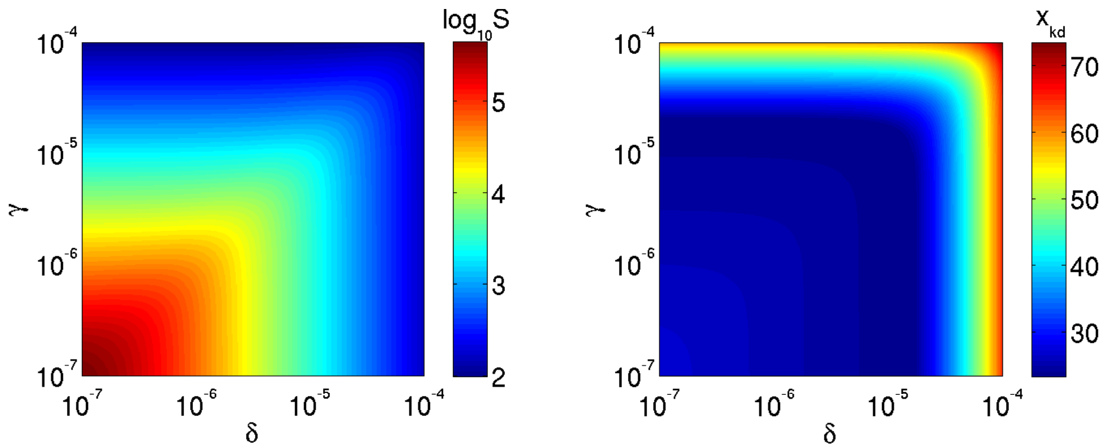

FIG. 3: Numerical illustration of the DM annihilation enhancement factor σ_0/σ_{nature} without considering kinetic decoupling effect (left) and the corresponding x_{kd} (right) on the $\gamma - \delta$ plane.

and T as Eq. (7), and use the formulae in Ref. [23] to calculate the T'_{kd}. We take a Z' model with $m_{DM} = 1$TeV as an example, but our results can be extended to other models (for more details, see the appendix). From our calculations, we find that for the typical parameters used to explain the PAMELA/Fermi/ATIC results $x_{kd} = m/T_{kd}$ is not far from $\tilde{x}_f \sim O(10)$. To show this point explicitly, we give the boost factor $S = \sigma_0/\sigma_{nature}$ (left) and x_{kd} (right) for different δ and γ in Fig. 3. Here $\sigma_{nature} = 3\times10^{-26}$ cm^{3}s^{-1} is the so called 'natural' value of DM annihilation cross section predicted by the WIMP models to generate correct relic density. In the left plot, we require each point in the parameter space producing the correct relic density without kinetic decoupling effect, and determine the corresponding σ_0. Then we use these δ, γ and σ_0 to calculate x_{kd}. We find in the parameter space favored by the PAMELA/Fermi/ATIC results with $S \sim O(10^3)$, x_{kd} is similar as $\tilde{x}_f$. It means the kinetic decoupling effect should be important in the early universe when determining the DM relic density. Therefore it should be considered carefully in the explanation of the anomalous cosmic positron flux.

IV. THE ENHANCEMENT FACTOR FOR ANOMALOUS POSITRON/ELECTRON FLUX

In this section, we calculate the enhancement factor by the Breit-Wigner resonance in the Galaxy today considering the kinetic decoupling. Here we define the enhancement factor as

$$S = \sigma_G/\sigma_{nature} \quad , \tag{16}$$

where σ_G denotes the $\langle\sigma v\rangle$ of DM with the most probable velocity $v_G \sim 10^{-3}$ in the Galaxy. This definition is different from the earlier form $s = \sigma_0/\sigma_{nature}$ [12, 13] as the DM velocity is not zero today. We will see such difference is important.

We give the numerical results of S in Fig. 4 and Fig. 5 for the cases of $\delta > 0$ and $\delta < 0$ respectively. For each point in the two figures, σ_0 has been adjusted to produce the correct relic density. The maximum value of S is only $O(10^2)$ with $\delta, \gamma \sim O(10^{-6})$. From these results, we find the Breit-Wigner enhancement is difficult to provide large enough boost factor to explain the anomalous positron excesses after taking into account the kinetic decoupling effect.

To check this result analytically, one would turn to the discussion in the last paragraph of Sec. II [12]. After the kinetic decoupling, the ξx^{-1} in Eq. (6) should be modified by $\xi x_{kd} x^{-2}$. The DM annihilation would continue to the temperature of $x_b \sim max[\delta,\gamma]^{-\frac{1}{2}} \cdot \sqrt{x_{kd}}$. One can also obtain an enhancement factor as $S \sim x_b/x_{f0} \sim max[\delta,\gamma]^{-\frac{1}{2}} \cdot \sqrt{x_{kd}}/x_{f0}$. It seems we could still achieve a required boost factor by taking some smaller parameters such as $(\delta,\gamma) \sim O(10^{-(6\sim8)})$. However, this is not the case. In fact, one can indeed obtain an arbitrary value of σ_0/σ_{nature} by setting the δ and γ tiny enough as discussed above. But the factor of σ_G/σ_{nature} is different with that in the vanishing DM velocity limit. From the Eq. (2) we can see, for the parameters of $\delta \simeq \gamma \geq O(10^{-5})$, these two factors are equal, because the $\langle\sigma v\rangle$ always reaches its maximum value σ_0 when the DM velocity decreases to $v_G \sim z \sim 10^{-3} < max[\delta,\gamma]^{\frac{1}{2}}$. However, for $\delta \simeq \gamma < O(10^{-6})$, $\sigma_G \sim \sigma_0(max[\delta,\gamma]/v_G^2)^2$ is

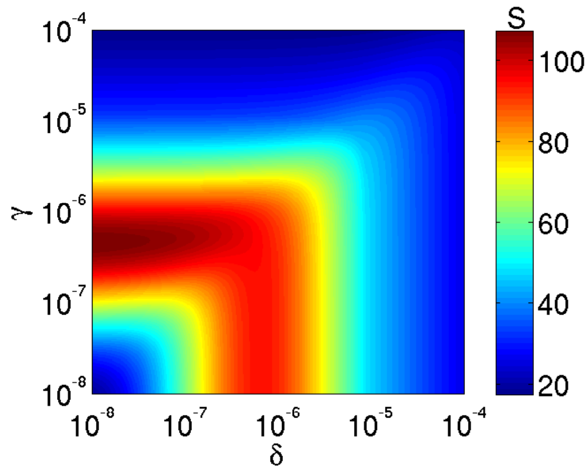

FIG. 4: Numerical illustration of the enhance factor of $S = \sigma_G/\sigma_{nature}$ on the $\gamma - \delta$ plane for the $\delta > 0$ case.

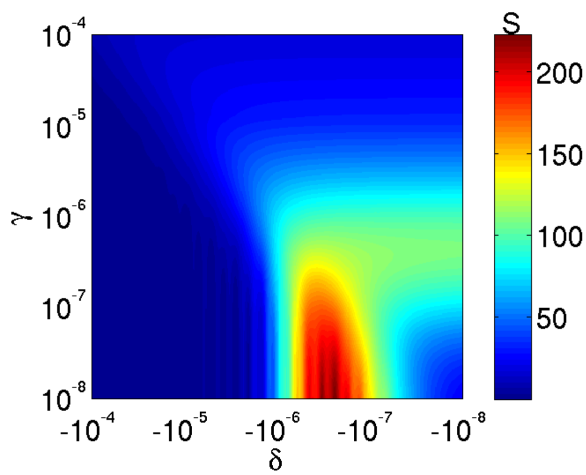

FIG. 5: The same as Fig. 4 but for $\delta < 0$.

always smaller than σ_0.

To understand the maximum value of σ_G/σ_{nature} and the corresponding parameters in Fig. 4 and Fig. 5, we can also use Eq. (2) as a roughly estimation. By setting $z \sim v_G^2 \sim 10^{-6}$ and $\sigma_0/\sigma_{nature} \sim max[\delta,\gamma]^{-\frac{1}{2}} \cdot \sqrt{x_{kd}}/x_{f0}$, we could obtain S

$$S \sim \frac{max[\delta,\gamma]^{\frac{3}{2}}}{max[\delta + v_G^2,\gamma]^2} \cdot \frac{\sqrt{x_{kd}}}{x_{f0}} \ . \tag{17}$$

From this rough estimation, we can see for the $\delta > 0$ case, there actually exists a maximum value of S around $max[\delta,\gamma] \sim v_G^2 \sim O(10^{-6})$ as shown in the Fig. 4. On the other hand, for the $\delta < 0$, when $\gamma \gg \delta + v_G^2 \to 0$, the S might be larger than the case of $\delta > 0$. It means at the physical pole resonance, if the annihilations in the galaxy occur accurately with $v^2/4 - \delta \to 0$, the cross section could be very large. However, considering the dispersion of the DM velocity our numerical results show the enhancement factor can not be very large either.

V. CONCLUSION AND DISCUSSION

In this work, we study the Breit-Wigner enhancement for DM annihilation taking the kinetic decoupling effect at the early universe into account. We find if the kinetic decoupling occurs at nearly the same epoch as the chemical decoupling, the DM annihilation process becomes very important and reduces the DM relic density significantly. Requiring the model gives correct relic density we find there is no parameter space that can give an annihilation cross section today large enough to explain the anomalous cosmic positron/electron excesses at PAMELA/ATIC/Fermi.

The main point here is the elastic scattering between DM and massless fermions $Xf \to Xf$ is not efficient to maintain DM in thermal equilibrium. The kinetic decoupling occurs at high temperature $\sim T_f$. The DM temperature would decrease as $\sim T^2/T_f$ after kinetic decoupling, and reaches a very small value before the structure formation. For typical WIMP such as neutralino, the typical damping mass is $\sim 10^{-6} M_\odot (m/100GeV)^{-\frac{3}{2}} (T_{kd}/30MeV)^{-\frac{3}{2}}$ [23]. Therefor in the Breit-Wigner enhancement with high T_{kd}, the damping mass might be much smaller than the usual cold DM model. This kind of DM model may predict tiny DM subhalo with $M_{sub} \ll 10^{-6} M_\odot$ in the Galaxy. The realistic impact for the structure formation in the Breit-Wigner mechanism may need a careful study. This feature is possible to change the predictions for DM indirect detection.

Finally we would like to point out that it is still possible to explain the anomalous cosmic positron excesses in some non-minimal Breit-Wigner models. The ideal here is adding some new interaction process to keep DM in kinetic equilibrium till to a low temperature. For example, the DM is slepton $\tilde{\tau}$ in the hidden sector [20]. It might interact with the hidden photon with a large coupling constant, or scatter with the standard model leptons by exchanging hidden neutralino in resonance. The hidden slepton annihilation to leptons could be enhanced by a Z' resonance in the $U(1)_{L_i - L_j}$ model [14]. With this setting to enhance the scattering process, it is possible to obtain a low T_{kd}, and re-

cover all the discussions in the earlier works about the Breit-Wigner mechanism.

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Appendix A: relation between cross sections of annihilation and elastic scattering processes

In this appendix, we would give detailed discussions about the relation between the cross sections of annihilation $X\bar{X} \to f\bar{f}$ and elastic scattering $Xf \to Xf$.

We assume the effective interaction Lagrangian between two DM particles (X) and two leptons (f) as

$$g_A g_B R_0 \bar{X}\Gamma_X X \bar{f}\Gamma_f f \tag{A1}$$

where g_A and g_B are interaction couplings of $X\bar{X}R$ and $f\bar{f}R$ respectively, Γ_X and Γ_f are combines of Lorentz metrics determined by model, R_0 is the propagator of resonance R. The cross section of annihilation process is (we neglect SM fermion mass m_f here)

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\sqrt{\frac{s}{s-4m^2}} g_A^2 g_B^2 |R_{0a}|^2 |M'_a|^2 \ , \tag{A2}$$

where we define the squared transition matrix element is $g_A^2 g_B^2 |R_{0a}|^2 |M'_a|^2$, s, t, u are usual Mandelstam variables. For annihilation process in the non-relativistic limit, s is approximated as $4m^2 + m^2 v^2$. So we can achieve σ_0 as

$$\sigma_0 = \frac{1}{32\pi m^2}\frac{g_A^2 g_B^2}{M^4(\delta^2+\gamma^2)}\int \frac{|M'_a|^2}{4\pi} d\Omega \ , \tag{A3}$$

For the cross section of elastic scattering, we find

$$\sigma v_s = \frac{1}{16\pi m^2}\frac{g_A^2 g_B^2}{M^4}\int \frac{|M'_s|^2}{4\pi} d\Omega \ , \tag{A4}$$

and the σv_s could be expressed by σ_0 as

$$\sigma v_s = 2\sigma_0(\delta^2+\gamma^2)\frac{\int |M'_s|^2 d\Omega}{\int |M'_a|^2 d\Omega} \ . \tag{A5}$$

In general, the $|M'_a|^2$ and $|M'_s|^2$ are expressed by the four vector momentums of four particles. We define p_1, p_2 for two DM particles, and k_1, k_2 for two fermions. We need only calculate either one of $|M'_a|^2$ and $|M'_s|^2$, and make some modifications to obtain the other one. In the calculation, we can neglect all the sub-leading terms which are proportional to m_f^2, v^2, and assumed the energy of fermion in the scattering process is $\omega = 3T/2$.

For example, we can calculate the relation between σv_s and σ_0 in a Z' model with $\Gamma_X = \Gamma_f = \gamma^\mu$. The $|M'_a|^2$ is given by

$$\begin{aligned}|M'_a|^2 &= \frac{1}{4}\cdot 32 \cdot [(k_1\cdot k_2)m^2 + (p_1\cdot k_1)(p_2\cdot k_2) \\ &+ (p_1\cdot k_2)(p_2\cdot k_1)] = 32m^4 \ .\end{aligned} \tag{A6}$$

Then we can achieve $\overline{|M'_s|^2} = 8m^2\omega^2$ and $\sigma v_s = \frac{9}{8}\sigma_0(\delta^2+\gamma^2)\frac{T^2}{m^2}$.

Appendix B: calculation for the kinetic decoupling temperature

In this appendix we show the calculation for the T_{kd} in a Z' model. The detailed method is described in Ref. [23], and can be extended to other models easily.

In general, the DM temperature $T_X(T)$ can be derived by solving Boltzmann equation

$$T_X = T\left[1 - \frac{z^{\frac{1}{n+2}}}{n+2} exp[z]\Gamma[-(n+2)^{-1}, z]\right] \ , \tag{B1}$$

where $z = \frac{a}{n+2}(\frac{T}{m})^{n+2}$. In the low (high) temperature limit $T \to 0$ ($T \to \infty$), the T_X has the same form $T_X \to T^2/m$ ($T_X \to T$) as Eq. (7). Then the kinetic decoupling temperature can be obtained as

$$T_{kd} = \left(\frac{T^2}{T_X}\right)_{T\to 0} = m\left[\left(\frac{a}{n+2}\right)^{\frac{1}{n+2}}\Gamma\left[\frac{n+1}{n+2}\right]\right]^{-1} \ , \tag{B2}$$

The parameters a and n are defined as follows. One need to expand the amplitude at $t = 0$ and $s = m^2 + 2m\omega$

$$|M|^2 = c_n\left(\frac{\omega}{m}\right)^n + O\left(\left(\frac{\omega}{m}^{n+1}\right)\right) \ , \tag{B3}$$

The constant a is given by

$$a = \sum_f \left(\frac{10}{(2\pi)^9 g_*}\right)^{1/2} g_f c_n N^{\pm}_{n+3}\frac{m_{pl}}{m} \ . \tag{B4}$$

The $N^{\pm}_{n+3}$ for fermion (plus sign) and scalar (minus sign) are given by

$$N_n^{\pm} = (1-p^{\pm}2^{-n})(n+1)!\zeta(n+1) \ , \tag{B5}$$

where $p^+ = 1$ and $p^- = 0$. For a Z' model with resonance mass $m_{Z'} = M \sim 2m$ described as

$$\frac{g_A g_B}{M^2}\bar{X}\gamma^\mu X \bar{f}\gamma_\mu f \ , \tag{B6}$$

we can achieve the amplitude at zero momentum transfer as $g_A^2 g_B^2 |M_s'|^2_{t=0}/M^4 = \frac{g_A^2 g_B^2}{2}(\frac{\omega}{m})^2$. Substituting $n = 2$ and $c_2 = g_A^2 g_B^2/2$ in the Eq. (B2), we obtain T_{kd}

$$T_{kd} = 1.326\left[\frac{\sqrt{g_*}m^5}{g_f c_2 M_{pl}}\right]^{\frac{1}{4}} \ . \tag{B7}$$

Here we assume the Z' has the same couplings with the different leptons and sum the g_f together. The thermal average annihilation cross section at low temperature can be written as $\sigma_0 = c_2/8\pi m^2(\delta^2 + \gamma^2)$, the Eq. (B8) can be re-written as

$$T_{kd} = 0.6\left[\frac{\sqrt{g_*}m^3}{g_f \sigma_0(\delta^2 + \gamma^2)M_{pl}}\right]^{\frac{1}{4}} \ , \tag{B8}$$

which is smaller than the result from Eq. (14) by a factor of O(1).

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